

Dynamics of Trusses Having Nonlinear Joints

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DYNAMICS OF TRUSSES HAVING NONLINEAR JOINTS

INTRODUCTION

The current Space Station design includes long beamlike lattices as the primary support structure as shown in Fig. 1. Two basic methods for lattice construction are under evaluation by NASA. The first uses erectable lattice members requiring astronaut EVA for construction while the second uses a pre-assembled but deployable truss requiring little EVA activity. One major disadvantage of deployable trusses is, however, the inherently nonlinear joints used in such structures. Usual analysis and testing techniques therefore become insufficient. The objective of this paper is to present an analysis technique that can perform the nonlinear static and dynamic analyses of a structure having nonlinear joints. Validation of the technique with test results still remains to be demonstrated.

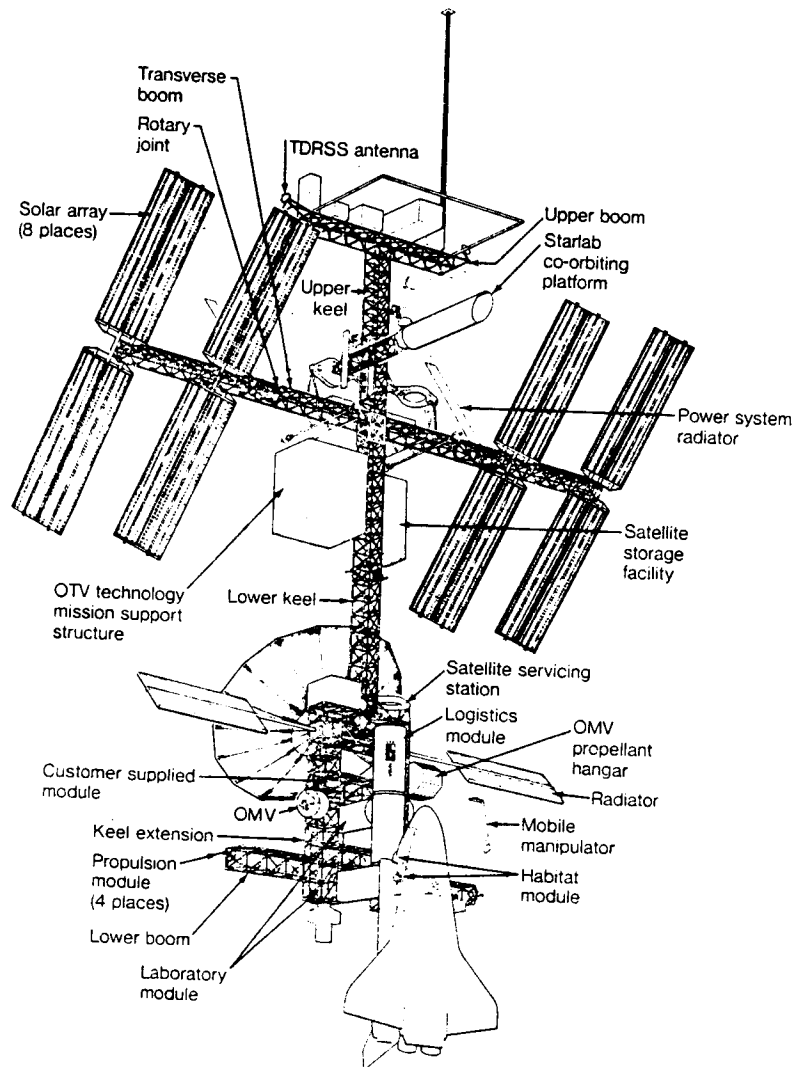


Figure 1

OBJECTIVES

The objectives of the transient analysis techniques are shown in Fig. 2. The technique must account for such nonlinear joint phenomena as free-play and hysteresis. The number of degrees of freedom (dof) describing the dynamic response of the structure must be reduced to a manageable size and the resulting equations of motion must permit a fast numerical algorithm. The analysis method must also permit inclusion of empirical data derived from tests on joints and truss "links". Finally, the analysis method must be validated by accurately predicting the response of a large multi-jointed test article.

Develop transient analysis techniques that can account for joint free-play and hysteresis

- Reduce the number of governing equations
- Develop a fast numerical integrator
- Validate using test results

Develop testing techniques that can identify and characterize the nonlinear effects of the joints

- Joint tests
- "Link" tests
- Truss tests

Figure 2

SOME NONLINEAR TRANSIENT ANALYSIS TECHNIQUES

Three different analysis procedures were examined rigorously in order to substantiate any particular result derived from one (see Fig. 3).

The "Gap-Element" approach currently exists in general finite element codes such as MSC/NASTRAN and ANSYS but this approach was found to be limited to problems having a small number of nonlinear joints. For large problems the Gap-Element approach became unstable. The general method used in the approach is to update the stiffness matrix each time the properties of a nonlinear element changed. For small problems this approach worked well, but for large problems, a small change in one nonlinear element caused "large" changes in all other elements and a unique stiffness matrix could not be found.

Perturbation techniques using the method of multiple scales were also examined for one and two dof problems yielding corroboration and insight into the behavior of spring-mass systems having free-play. The technique requires, however, an enormous amount of algebraic complexity and may be too limited for an analytic description of trusses having arbitrary joint nonlinearities. Nevertheless, the multiple scale technique remains a valuable tool for future research.

The technique developed in this paper is coined the "residual force" technique. In this method, the linear and nonlinear character of the structure are separately identified and placed, respectively, on the left and right hand sides of the equations of motion. The residual forces appearing on the right hand side (RHS) represent the nonlinear corrections that must be applied to a linear structure in order to replicate the nonlinear response. Having the linear terms on the left hand side (LHS) permits powerful modal analysis techniques as a viable method of size reduction.

Results using the residual force technique have been shown to agree with the results using the gap element and multiple scale solutions for small problems. For large problems, no other known technique exists.

"Gap-element" approach

- Slow—stiffness matrix updated each time step
- Unstable for large problems

Residual force approach

- "Left hand side" of equations of motion linearized
- Nonlinear joint phenomena on "right hand side"

Perturbation techniques

- Method of averaging
- Multiple scale technique

Figure 3

BASIC IDEA OF THE RESIDUAL FORCE

The basic idea of the residual force can best be described by examining the simple one-dof problem shown in Fig. 4. The joint in this spring-mass system is modeled simply as a gap having a total free-play of 2δ . The force-displacement curve for this system therefore has a flat spot with zero force while in the gap. This curve can also be reproduced by including a small residual force acting on a linear spring having no gap. The equations of motion then take the form as shown in Fig. 4. Note that the stiffness derived on the "LHS" can be derived by considering the joint to be infinitely stiff.

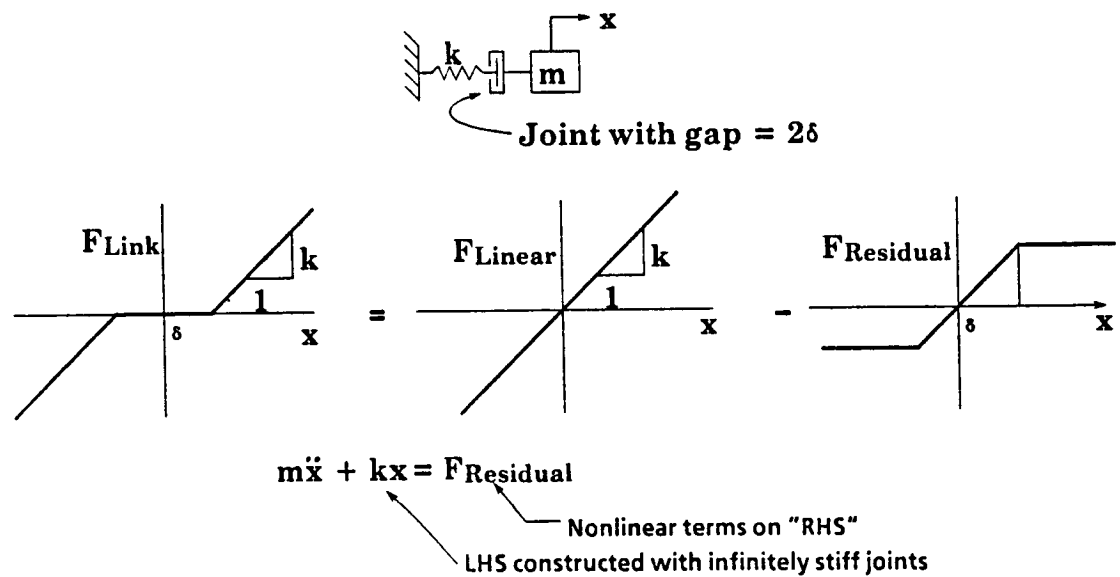
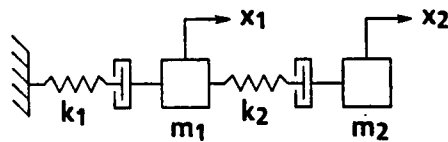


Figure 4

TWO DOF PROBLEM

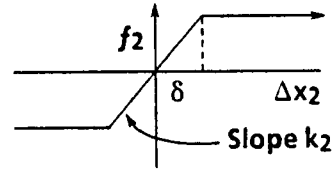
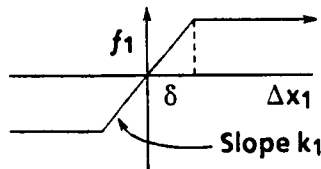
The equations of motion for a two dof gap problem are shown in Fig. 5. The system consists of two springs, two gaps, and two masses as shown. The linear stiffness matrix of the "LHS" is again constructed by considering the joints to be infinitely stiff, that is, by assuming the gaps to be locked. The nonlinear effects of the gaps are defined by two residual forces appearing on the "RHS" of the equations of motion. Note in Fig. 5 that both the nonlinear forces acting on the system and the relative displacements across each spring-gap element are each defined by the same transfer matrix that depends only upon the geometry and connectivity of the structure.

Results will now be presented for this two dof problem in order to demonstrate certain nonlinear effects that gaps can have on the dynamic response. Afterwards, the equations of motion for a joint dominated structure having arbitrary joint nonlinearities will be presented along with accompanying results for a 10 bay deployable truss.



Equations of motion:

$$\begin{bmatrix} m_1 & \\ & m_2 \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} k_1 + k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$



$$\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Figure 5

MODAL RESPONSE OF THE TWO-DOF PROBLEM

The nonlinear modal response of the two dof problem subject to an initial impulse is shown in Fig. 6. The modal response of the linear system having no gaps is also shown. Two effects that gaps have on the modal response are immediately apparent when the linear and nonlinear solutions are compared. First, the maximum modal amplitudes for the nonlinear problem are larger. And second, the modal periods for the gapped system are longer than the periods for the gapless system. Both of these effects are understandable in that the gaps soften the structure and a softer structure would respond with larger amplitude and increased period when excited with the same initial impulse. Other nonlinear effects become apparent, however, when the solution is viewed for longer time periods.

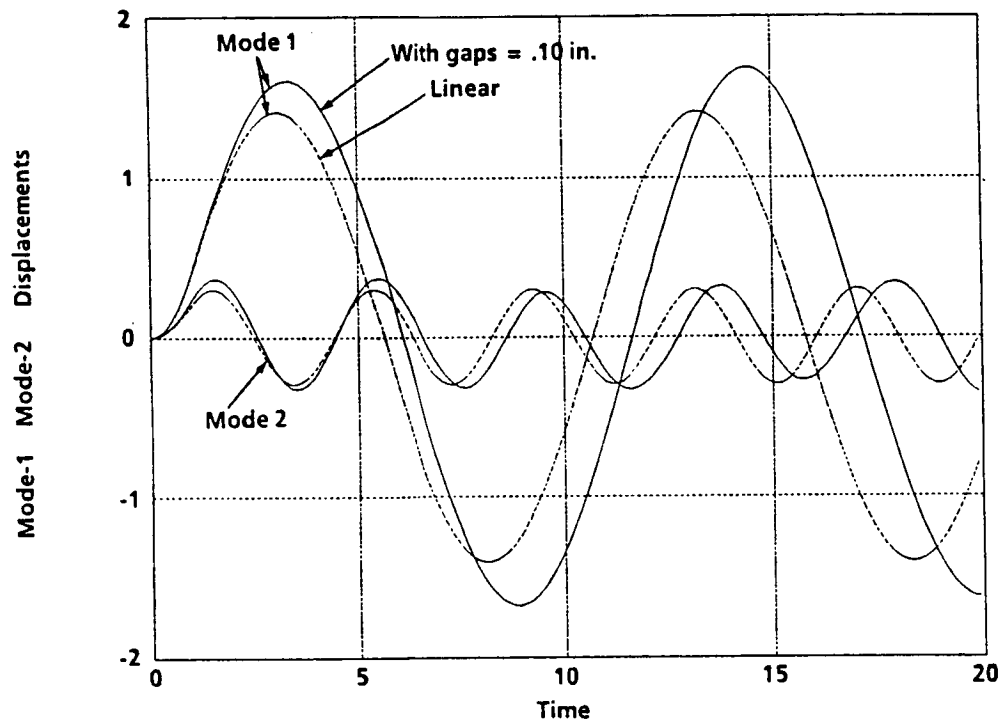


Figure 6

UNDAMPED TWO-DOF PROBLEM

The modal responses of the undamped two-dof gap problem are shown for an extended period of time in Fig. 7. Nonlinear coupling between the modes now becomes apparent in that the free undamped vibration of the two modes exhibit slow sinusoidal variations in their amplitudes. Note that an increase in the maximum amplitude of the second mode is accompanied by a decrease in the amplitude of the first mode. This reciprocal variation in amplitude is understandable since the total energy of the system must remain constant after the initial impulse. The amplitude variations thus indicate a slow sinusoidal energy transfer back and forth between the two modes.

The slow sinusoidal variations in the modal amplitudes may also be shown to exist by using the perturbation theory of multiple-scales on the two-dof gap problem.

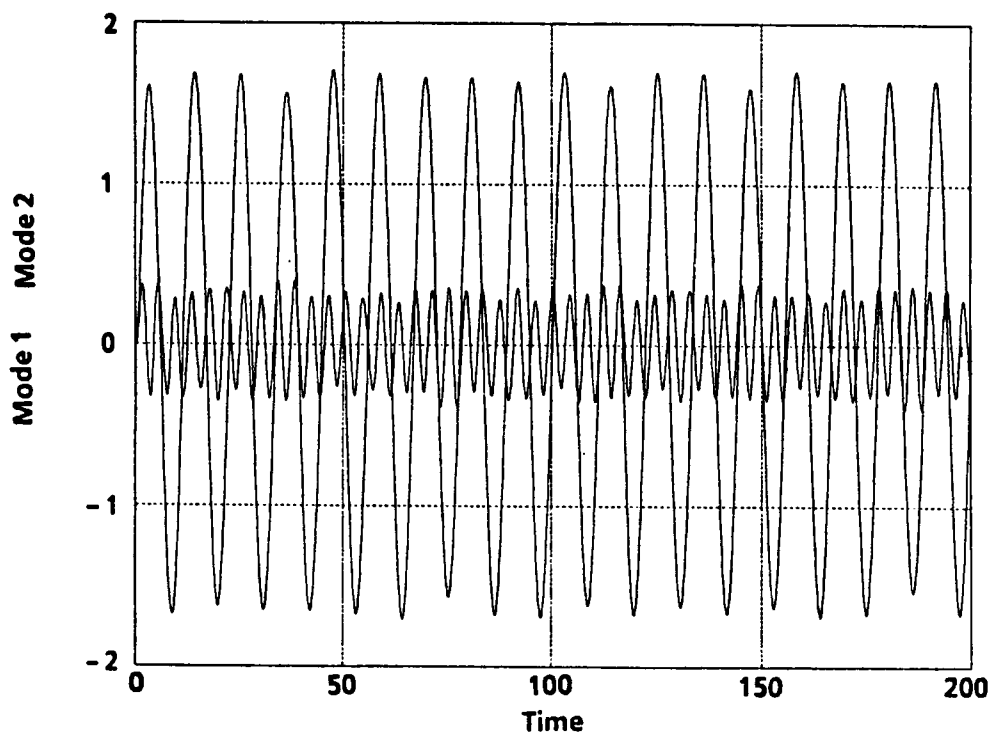


Figure 7

TWO-DOF PROBLEM WITH .5% MODAL DAMPING

The effect of energy transfer between the two modes becomes important when damping is introduced. Fig. 8 shows the response of the two-dof gap problem due to an initial impulse and 0.5 percent modal damping. Slow sinusoidal variations in amplitude are again apparent, but because the second mode has a greater exponential decay, the energy that is transferred from the first mode to the second mode cannot be equally returned. The result is that the second mode seems to reach a quasi-steady state response with the energy dissipation due to modal damping being balanced by the energy transfer from mode one. This also means that the first mode will appear to damp faster than would otherwise be predicted from modal damping alone.

Note also that the amplitude of the second mode is given a substantial boost whenever the two modes are in phase. This suggests that there may be certain conditions determined by the ratio of the two natural modal frequencies and the gap sizes that may cause internal resonance. A complete investigation into the effects that gaps may have on the response of a system must therefore seek to define any conditions that may lead to internal or parametric resonance between the modes.

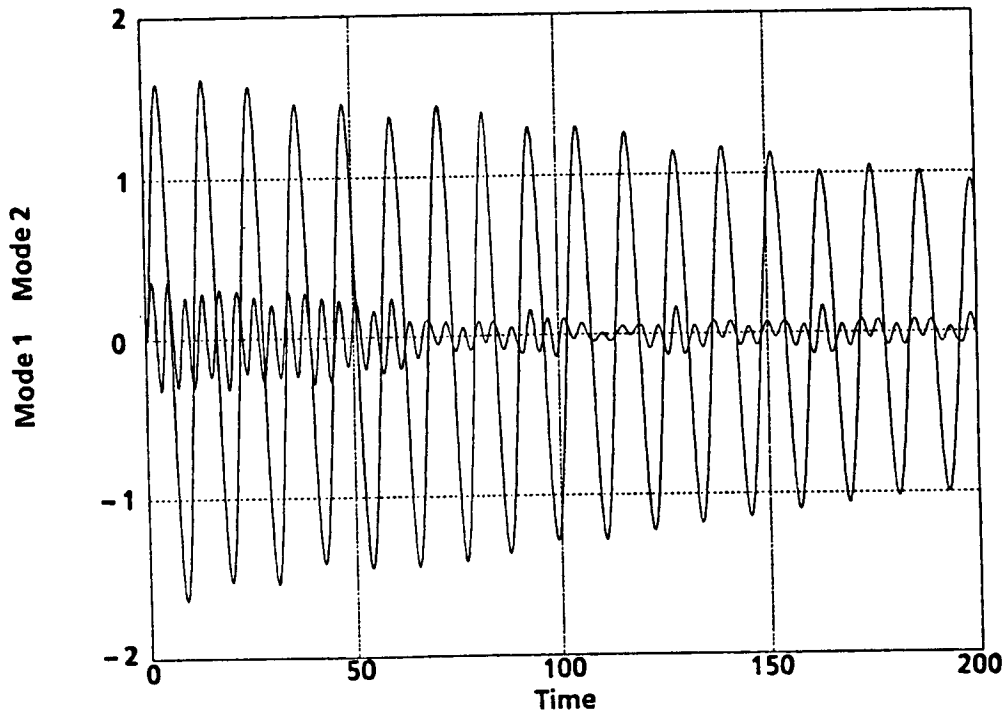


Figure 8

TWO-DOF PROBLEM WITH ENHANCED COUPLING

The coupling between the modes of the two-dof gap problem can be enhanced by a proper choice of the modal frequencies. Forced response of the single dof gap problem shows that resonance can occur when the natural frequency is $1/3$, $1/5$, $1/7$, etc. of the driving frequency. This suggests that choosing the frequency of the second mode to be three times the frequency of the first should define a problem exhibiting large coupling between the two modes. Fig. 9 shows the response of an undamped two-dof gap problem when the ratio of frequencies between the first and second mode is $1/3$. This is precisely the condition that one would expect to see resonance of the second mode if such resonance does in fact exist. Fig. 9 shows, however, that while large non-sinusoidal variations do occur in the free vibration of the second mode, time linear growth in the amplitude does not occur. Nevertheless, the slow sinusoidal variations in the amplitude of the second mode are no longer small and indicates a greater coupling between the first and second modes.

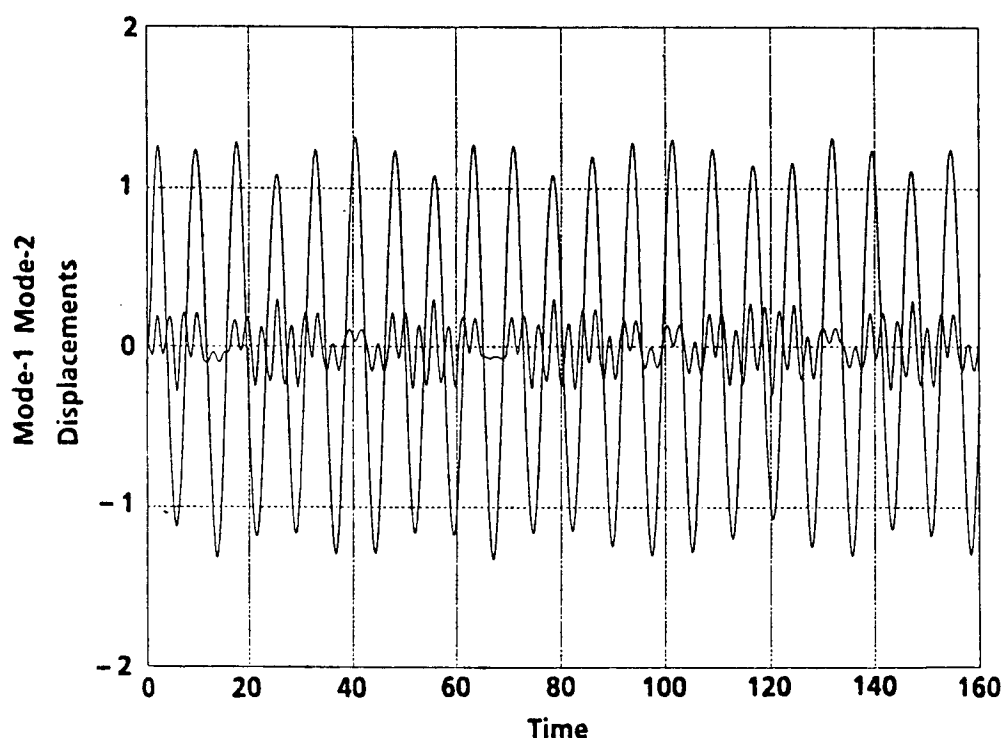


Figure 9

MODELING ASSUMPTIONS

The modeling assumptions used by the residual force technique in the analysis of a typical deployable truss are shown in Fig. 10. As shown, the longerons and lacing members have two or three nonlinear joints that can be characterized using force maps. A force map of a joint is simply a pictorial way of stating that the force in a joint can be defined as a function of both the relative displacement and velocity across the joint. Note that the battens are shown not to have nonlinear joints. This is because stable behavior of trusses (or beamlike lattice structures) generally require that all battens be rigidly attached to the lattice vertices. If the battens are pinned instead of rigidly attached, geometric nonlinearities due to the finite size of the joint must be considered. Moreover, low frequency joint rotation modes will exist unnecessarily complicating the dynamic behavior of the structure. Deployable trusses should therefore avoid pinned battens if at all possible.

Another modeling assumption required in the residual force technique is that the mass of the truss can be lumped at nodes. This approximation is usually valid for low frequency excitation as is generally the case for the Space Station. It remains to be seen if damping effects can be accurately calculated when using the lumped mass approach.

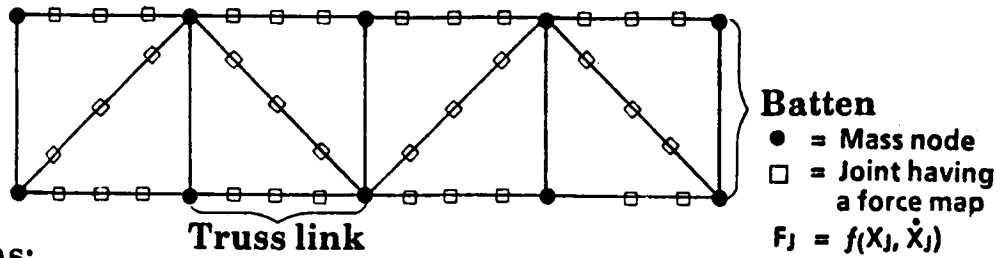
The concept of a truss link is also pictorially shown in Fig. 10. A truss link is defined here as the composite series of joints and members that represent the truss structure between two truss vertices. Truss links are ideally considered as axial load carrying members only and are modeled as a series combination of nonlinear joints and linear springs.

The complete description of the truss link requires, in general, monitoring all the "internal" dof of the link that describe the relative displacements of each joint and spring. In certain special instances however, a composite force map for a massless truss link can be derived. First, if all joint force maps depend only upon displacement then an equivalent force map for the link can be easily derived. Second, if the massless truss link has only two arbitrary but identical joints then a residual force map for the link can be derived. And finally, if the joint stiffness is large, the damping small, and the rates low for each joint, then the force map for the link can again be derived. This last special instance is generally the case for Space Station trusses and suggests that an equivalent force map for the link can be derived directly from testing. If none of the above three special instances apply to the truss being analyzed, then all interior dof of the link must be monitored during the analysis. One easy way to accomplish this is to simply include additional mass freedoms along the truss link.

Special attention has been given to the modeling of the truss links because the success or failure of a transient analysis technique strongly depends upon the ability to accurately monitor the nonlinear stiffness and damping effects of the generally stiff joints. Direct monitoring of the extremely small relative displacements across the joints is impractical. Instead, the residual force method takes advantage of the fact that the joints are in series with a relatively soft spring and a residual force map for the link is derived. In essence, the forces in the joints are monitored instead of the relative displacements.

Dynamics of Trusses

Modeling Assumptions



Assumptions:

- Battens do not have pinned joints (otherwise joint rotation modes would exist)
- Truss links are axial load carrying members only
- Joints are described by arbitrary force maps
- Inertial effects can be lumped at the mass nodes

Claim:

- The ability or inability to analyze the above truss is determined by the ability or inability to analyze the nonlinear "truss links" with an efficient, stable numerical integrator

Figure 10

RESIDUAL FORCE MAPS

The derivation of a residual force map for a joint in series with a soft spring is given in Fig. 11. The force in the joint can be described by a force map giving the joint force as an arbitrary function of the relative displacement and velocity across the joint. This force must also equal the force in the soft spring, and both the spring force and the joint force are equal to the force in the link.

The first step in generating the residual force map for the link is to transform the displacement axis of the joint force map so that the new force map is a function of the total link displacement and the joint's relative velocity. The equations used in this first step are shown in Fig. 11.

The second step uses the definition of the residual force as the difference between the linear "left hand side" force and the total nonlinear link force. Note that the linear force in Step 2 of Fig. 11 is again obtained by considering the joint to be infinitely stiff. As a result of the above definition for the residual force, it is found that the joint's relative displacement is directly proportional to the residual force. The second step thus transforms two axes of the force map from step 1; the force axis is transformed into a residual force axis, and the velocity axis is transformed to a new axis having the time derivative of the residual force as the independent variable.

The main advantage of the above transformations is that the incrementally small joint displacements and velocities are not monitored directly. Instead, a very small and stretched out residual force map is used offering a numerically more attractive description of the link.

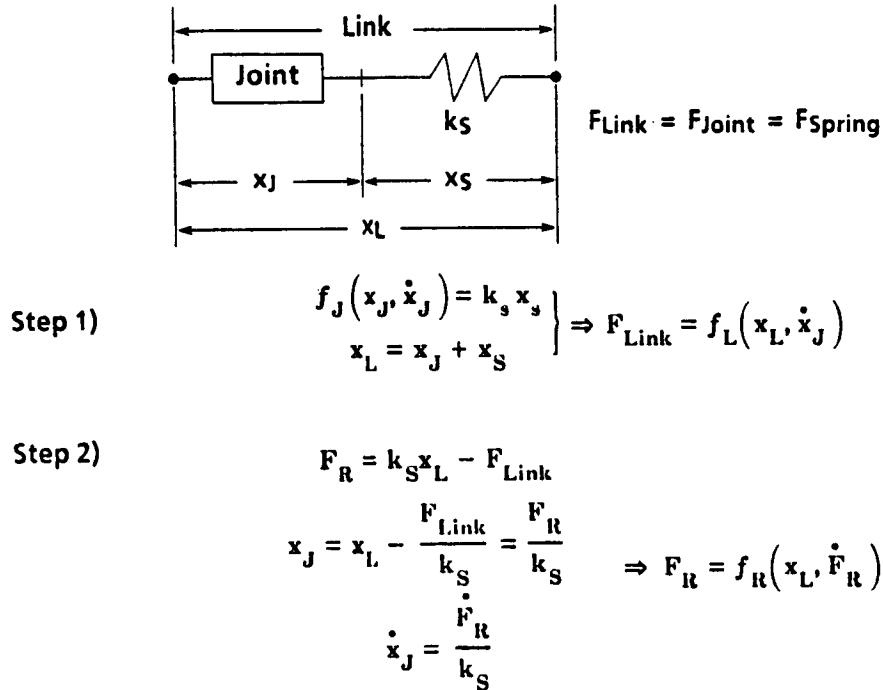


Figure 11

RESIDUAL FORCE MAP EXAMPLES

Two examples of residual force maps are given in Fig. 12. The residual force map for a gap in series with a soft spring was derived in Fig. 4 and is shown here again to demonstrate that the residual force for this problem is expressed in terms of the total relative displacement across the link. This result will also be true for any number of joints in series with a soft spring so long as the force maps of the joints are independent of velocity.

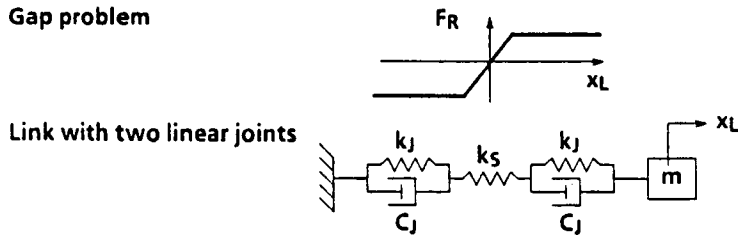
The second example is of two identical Voigt joints in series with a soft spring. The truss link is also grounded at one end and attached to a mass at the other to formulate a single dof problem so that all equations of motion may be shown.

Using the procedure of Fig. 11 for calculating the residual force, a linear first order differential equation for the residual force is derived. The equations of motion for the link-mass system are also derived by considering the Voigt joints to be infinitely stiff. All equations are shown in Fig. 12.

There are two interesting observations to be made about the first order differential equation for the residual force. First, the derivative term is normally small suggesting a perturbation solution to the differential equation. And second, the nonhomogeneous term on the right hand side of the differential equation is always small for joints that are much stiffer than the "soft" link spring. Monitoring the residual force therefore appears to be much more numerically attractive than monitoring the incrementally small displacements and velocities across the Voigt joints.

The perturbation solution of the first order differential equation for the residual force also gives an interesting result. As shown in Fig. 12 the perturbation solution for the residual force can be expressed as a function of the link's relative displacement and velocity. This means that the residual force for the link is itself expressible in terms of a force map. This result will always be true whenever the joint stiffness is large, the joint damping is small, and the rates are low.

Gap problem



Link with two linear joints

$$m\ddot{x}_L + k_S x_L = F_R$$

$$\frac{C_J}{k_J + 2k_S} \dot{F}_R + F_R = (k_S - k_L) x_L \quad \frac{1}{k_L} = \frac{1}{k_S} + \frac{2}{k_J}$$

$$\text{Also } F_R \approx (k_S - k_L) x_L - \frac{(k_S - k_L)}{k_J + 2k_S} C_J \dot{x}_L = f(x_L, \dot{x}_L)$$

when damping is low, joint stiffness is large, and rates are low

EQUATIONS OF MOTION OF A TRUSS HAVING NONLINEAR JOINTS

The equations of motion governing the free and forced dynamic response of a truss having nonlinear joints are shown in Fig. 13. These equations were derived using the residual force technique on a truss satisfying the assumptions listed in Fig. 10. One essential feature of this technique is to replace the arbitrary force maps describing the nonlinear joints with residual force maps describing the truss links. The main advantage of this replacement is that the incrementally small relative displacements and velocities across a joint are not monitored directly thereby avoiding numerical difficulties. Instead, very small and "soft" residual forces are defined giving a numerically attractive form for the equations of motion and thereby permitting numerically stable integration algorithms.

The only mass degrees of freedom shown in Fig. 13 are at the truss vertices but additional mass freedoms along each truss link may be required depending upon the nature of the joint nonlinearities as discussed under the topic of modeling assumptions (Fig. 10). The total number of degrees of freedom defined by the nodal equations of motion shown in Fig. 13 can be on the order of 2000 degrees of freedom for Space Station trusses and methods to reduce this large number are therefore desired.

The modal representation also shown in Fig. 13 is a natural choice for size reduction in that it takes advantage of the linearity of the left hand side of the nodal equations of motion. However, using a truncated set of structural modes generally has the disadvantage of decreasing the represented flexibility of the structure. This disadvantage can be offset by including the residual flexibility due to the neglected modes in all calculations affecting the dynamic response of the structure. The links' relative displacements and velocities therefore have residual flexibility terms to augment the modal descriptions. Using a truncated set of system modes in the equations of motion then only assumes the inertial loads due to the neglected modes can be ignored. The number of system modes to be retained must therefore be chosen with care.

The residual flexibility matrix operating on the residual forces will in general be very large. Practical inclusion of this matrix is then only possible when the matrix is nearly diagonal. This is expected to be generally the case for trusses but has not yet been demonstrated. However, for problems considered to date, the flexibility terms have not been required. The numerical accuracy of the results were determined simply by including most if not all of the system modes and comparing the results to the truncated solution. Future research will concentrate on the number of retained modes versus residual flexibility issue for various joint nonlinearities.

The two main types of joints investigated so far have been the nonlinear gap joint and the Voigt joint. The Voigt joints considered consist of a stiff spring in parallel with either a lightly damped or heavily damped dashpot. Successful inclusion of these two main types of joints in the transient analysis of a large joint dominated truss should demonstrate the general capability of the residual force approach. Results using these joints are included here for a four bay planar truss and a ten bay 3D deployable truss currently at MARSHALL.

Dynamics of Trusses

Residual Force Technique

Equations of motion

$$M\ddot{x} + Kx = CF_R + F_{\text{External}}$$

x = displacements at mass freedoms only

$x_L = CTx$ = deflections across a link

F_R = residual forces in links = $f(x_L, \dot{F}_R)$

Modal representation

$$\ddot{Q} + \omega^2 Q = \Phi_L^T F_R + \Phi^T F_{\text{External}}$$

$$x_L = \Phi_L Q + G_L F_R + G_E F_{\text{External}}$$

$$\dot{x}_L = \Phi_L \dot{Q} + G_L \dot{F}_R + G_E \dot{F}_{\text{External}}$$

$$G_L = CTG_R C \quad G_E = CTG_R$$

$$G_R = (K^{-1} - \Phi 1/\omega^2 \Phi^T) = \text{residual flexibility}$$

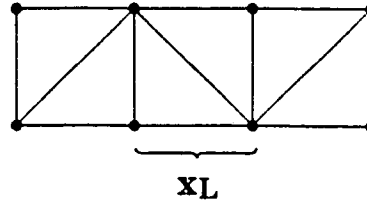


Figure 13

FOUR BAY PLANAR TRUSS

The truss model of a four bay planar truss used to perform several numerical experiments is shown in Fig. 14. Gaps or Voigt joints were considered in all longerons and lacing members but were not included for the battens.

The response after an initial impulse for a truss having gaps of 0.003 inches is shown in Fig. 15 at two different times. The response of the linear truss is also shown for comparison. The nodal equations of motion were integrated so that all modal excitations would be included. Two observations can be made by examining Fig. 15. First, the maximum amplitude of the gapped structure at time 0.270 seconds is larger than the maximum amplitude (nearly) of the linear structure. The larger amplitude appearing for the gapped response results because the gaps introduce greater flexibility to the structure. And second, the nonlinear response has a much greater modal participation than the linear response as seen by observing the structural deformation at time 0.540 seconds.

The tip and modal responses of the gapped truss having 1% modal damping are shown in Fig. 16. The initial large amplitudes in these responses is due to the initial impulse applied to the structure. Free vibration then occurs for times greater than 0.5 seconds. Several interesting observations can be made by examining Fig. 16. First, the tip response is governed primarily by the first mode. Second, as the amplitude decreases due to modal damping, the period increases. Third, very small, slow sinusoidal variations occur in the amplitude of the first mode but these variations are not as pronounced as seen earlier for the two-dof gapped problem. Fourth, the free vibration response of the second mode appears to be strongly coupled to the first and higher modes. Coupling to the first mode can be inferred due to the quasi-steady state response that occurs between four and ten seconds. The energy dissipation that this mode should normally display has been balanced by the net energy transferal from the first mode. Coupling to the higher modes can be inferred by the high frequency content in the free vibration for this mode.

The tip and modal responses of the four bay planar truss having Voigt joints are shown in Fig. 17. The response of this structure to an initial impulse is particularly interesting because all of the damping present is due to the Voigt joints. Modal damping is not present. Several observations concerning this response are also possible, particularly when also compared to the response of Fig. 16. First, the tip response is again dominated by the first mode. Second, the contribution of the second mode is much smaller for the Voigt jointed model than it is for the gapped model. And third, frequency shifts and slow sinusoidal variations do not occur in the decaying response as they do in the gapped problem.

Dynamics of Trusses

2-D Truss With Sloppy Joints

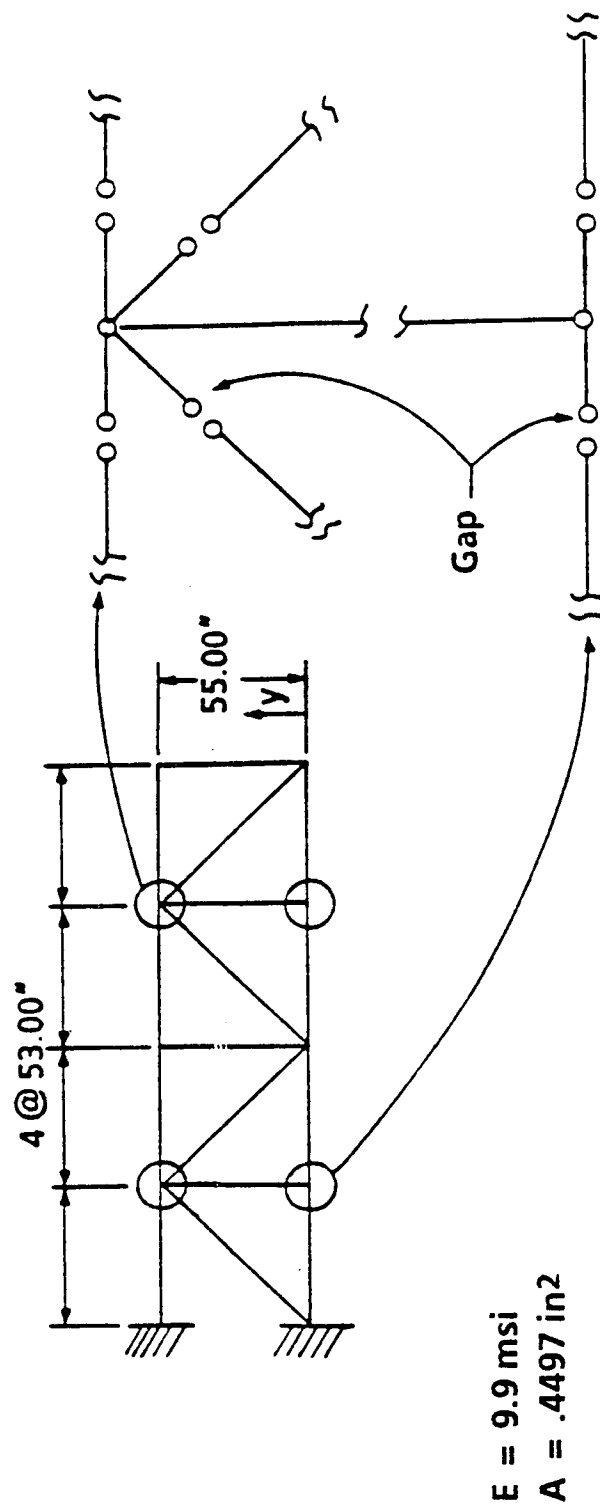


Figure 14

2-D Truss Results

Residual Force Approach

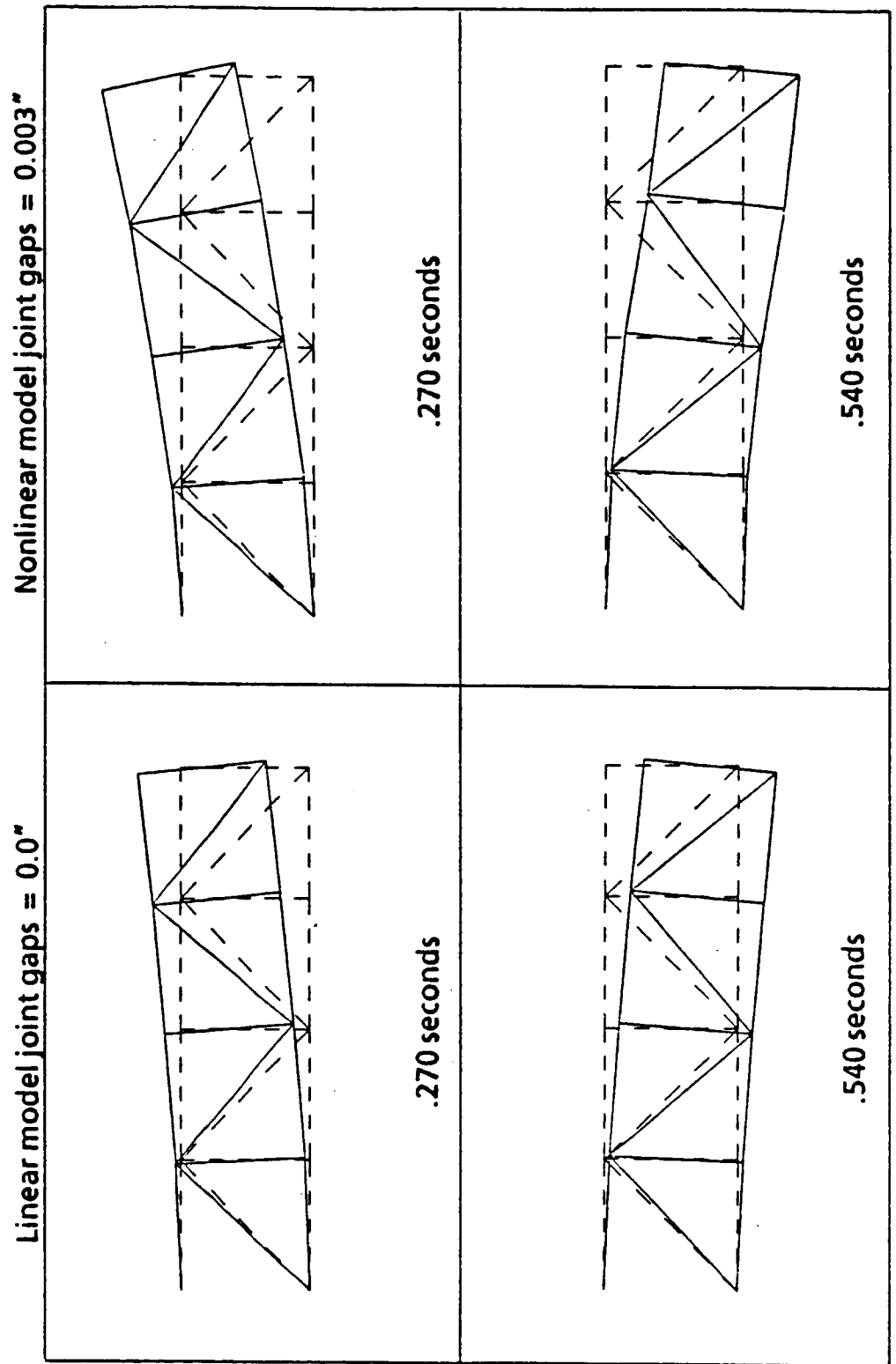


Figure 15

2D Rockwell Truss

Gaps = .003 Inches, 1% Modal Damping

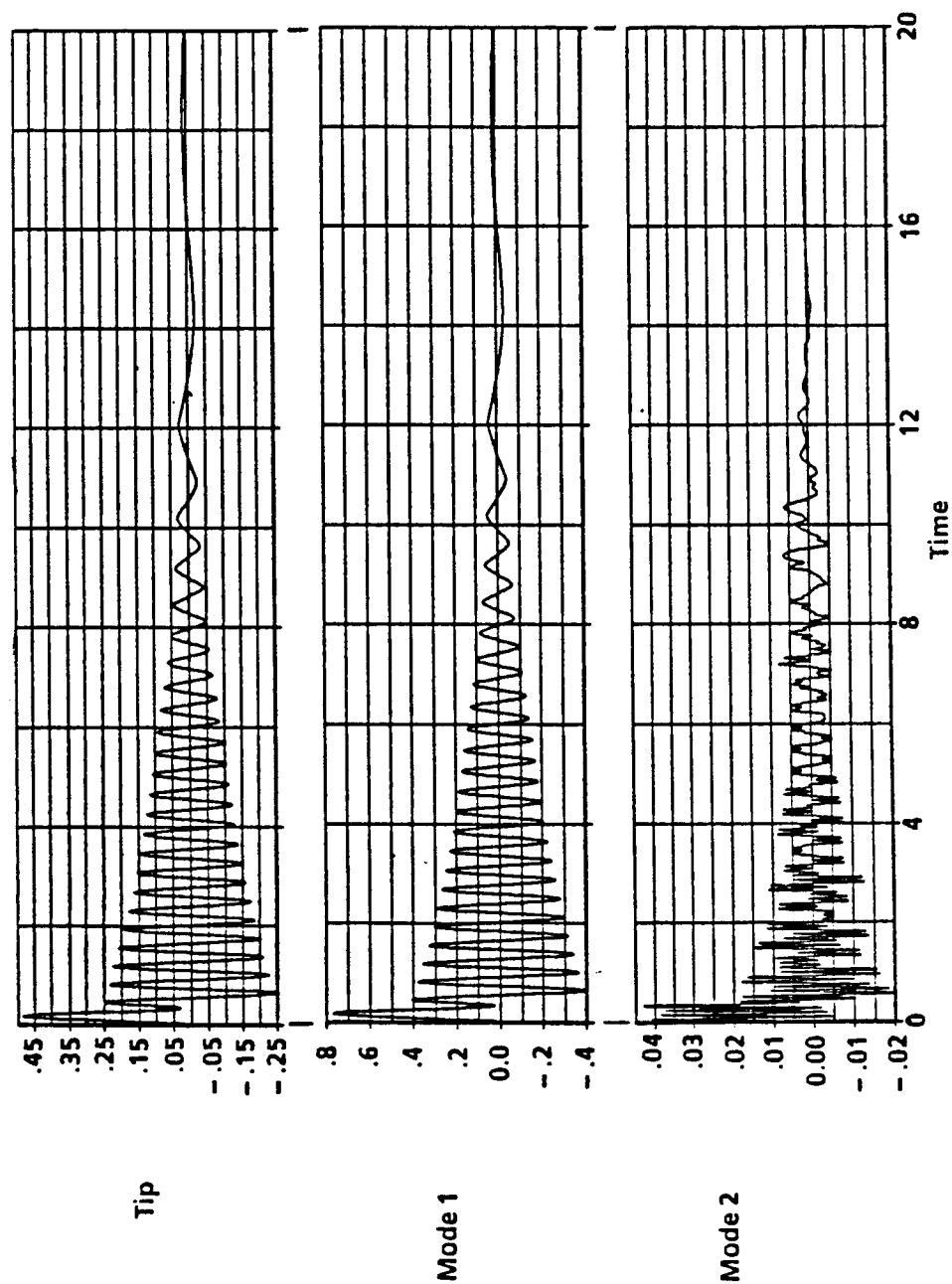


Figure 16

2D Rockwell Truss

Joints = Spring and Dashpot, Damping from Joints Only

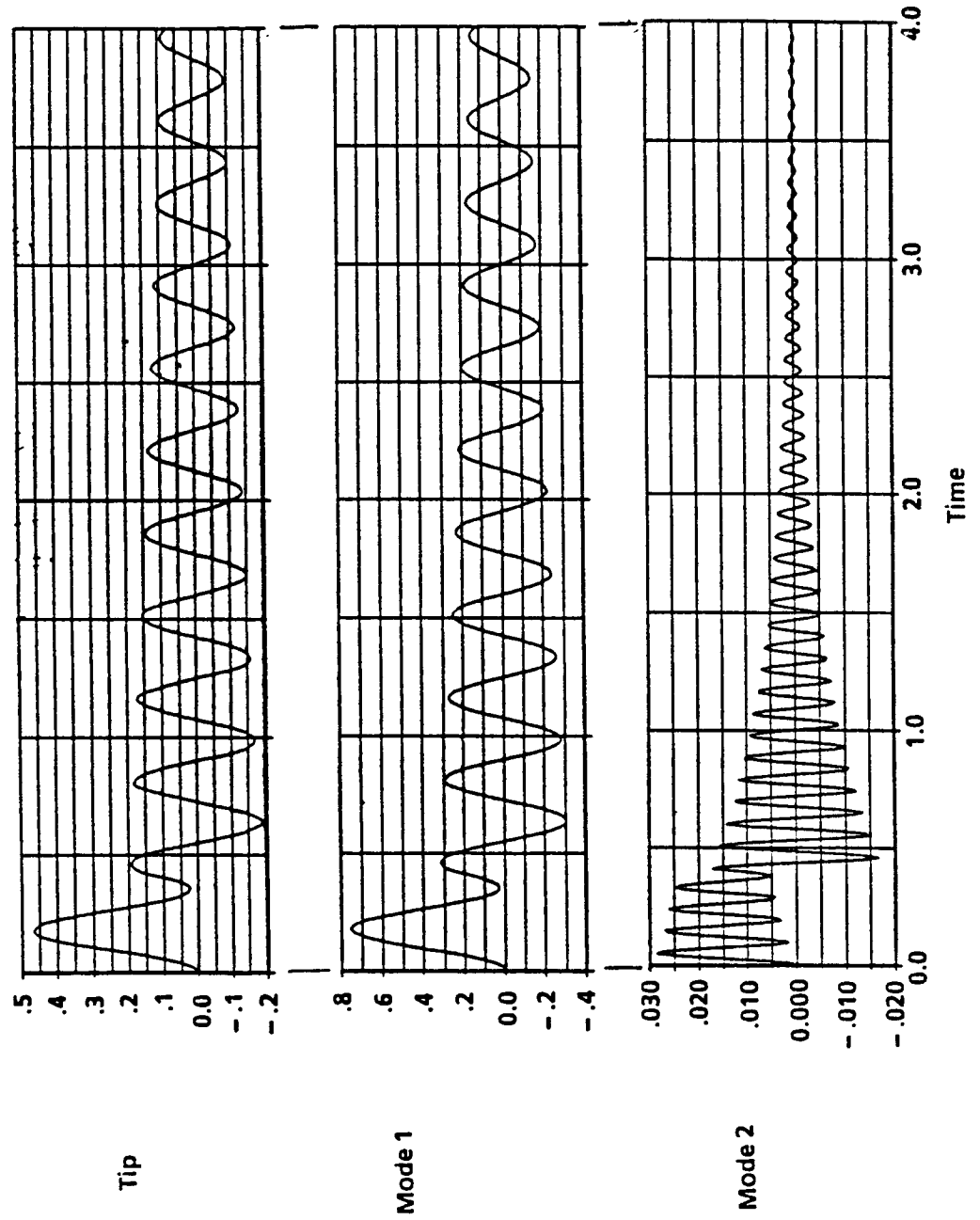


Figure 17

TEN BAY ROCKWELL TRUSS

A nonlinear transient analysis is also performed for a ten bay deployable truss that was designed and constructed by Rockwell and is awaiting testing at MARSHALL. Fig. 18 shows the first and second bending modes for this truss where each bending mode actually represents two orthogonal modes having identical frequencies. Gaps of 0.004 inches were included in all the longerons and lacing links. This gap value is reasonable in that each link has three deployable joints. The longeron links have two pin joints and one hinge joint, and the diagonal links have two pin joints and one telescoping joint.

The tip response of the gapped ten bay cantilevered truss having 1% modal damping subject to an initial impulse is shown in Fig. 19. The response of the linear gapless structure is also shown for comparison. Three observations can be made from Fig. 19. First and second, the amplitude and period of the nonlinear response is greater than those for the linear structure. The most interesting observation, however, is that the damping of the nonlinear structure appears to be greater than 1 percent. Evidently, energy is being transferred from the lower to the higher modes as a result of the nonlinear coupling between the modes. This phenomena was seen earlier for the two-dof problem.

The nonlinear coupling between the modes is clearly shown in Fig. 20. Nearly equal response in modes 1 and 2 as well as in modes 4 and 5 is due to the fact that the initial impulse excited these modes equally. The decaying response of modes 1 and 2 again show the phenomena that the period increases as the amplitude decreases. The response of modes 4 and 5, however, does not appear to be decaying exponentially as expected for modal damping. A strong 2 hertz component in modes 4 and 5 indicates strong coupling with the first bending modes and offers an explanation why decay is not also occurring for the second bending modes. Modes 1 and 2 are evidently driving the response of modes 4 and 5 with sufficient intensity to overcome damping. A net energy drain from modes 1 and 2 to the higher modes will therefore result. This phenomena also explains why the modal damping of modes 1 and 2 seems to be larger than the allotted 1 percent, the difference being made up by the energy transferal to the higher modes having a greater energy dissipation potential.

Fig. 21 shows the linear and nonlinear responses for mode 4. Note that the maximum response occurs shortly after the initial impulse and that the magnitude of the nonlinear response is much greater than the magnitude of the linear response. Having gaps in the truss therefore permits greater modal participation for the applied tip loading impulse. Note also that if no coupling between the modes were to exist, then mode 4 would decay relatively quickly.

3D Rockwell Truss

10 Bays with 780 lb Tip Mass

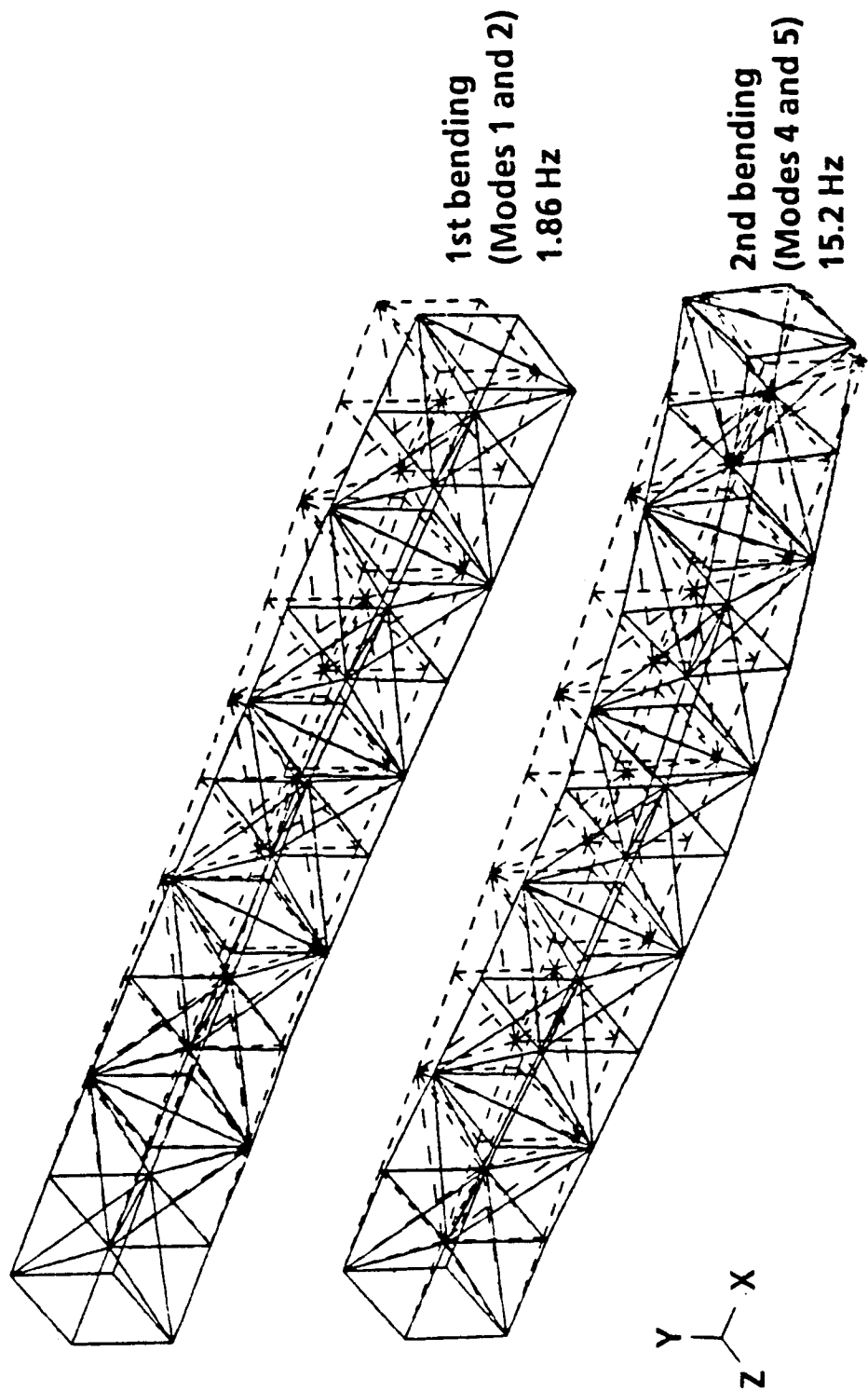


Figure 18

3D Rockwell Truss – Linear Vs Nonlinear

1% Model Damping

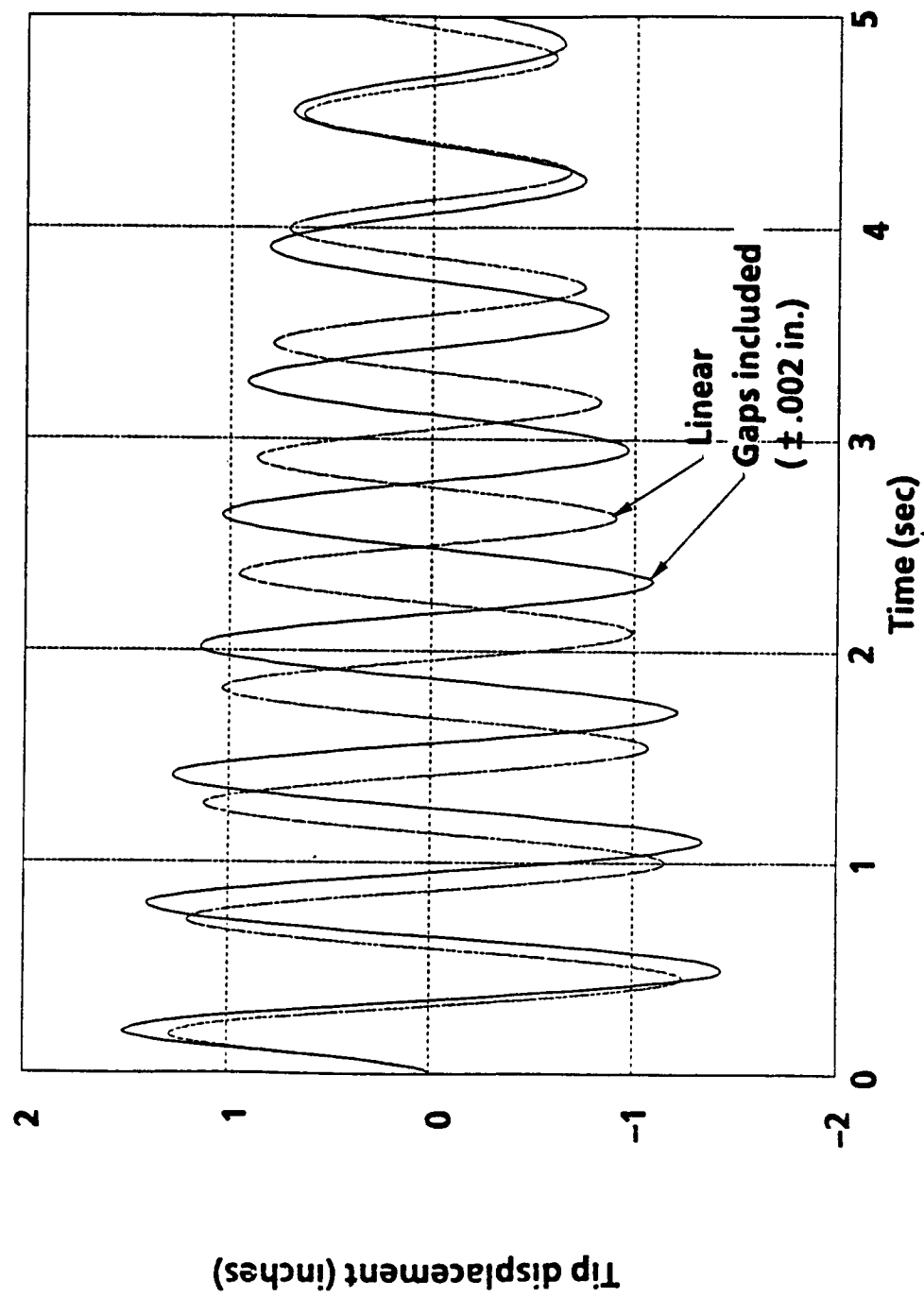


Figure 19

3D Rockwell Truss with 1% Modal Damping

Link Gap = $\pm .002$ in

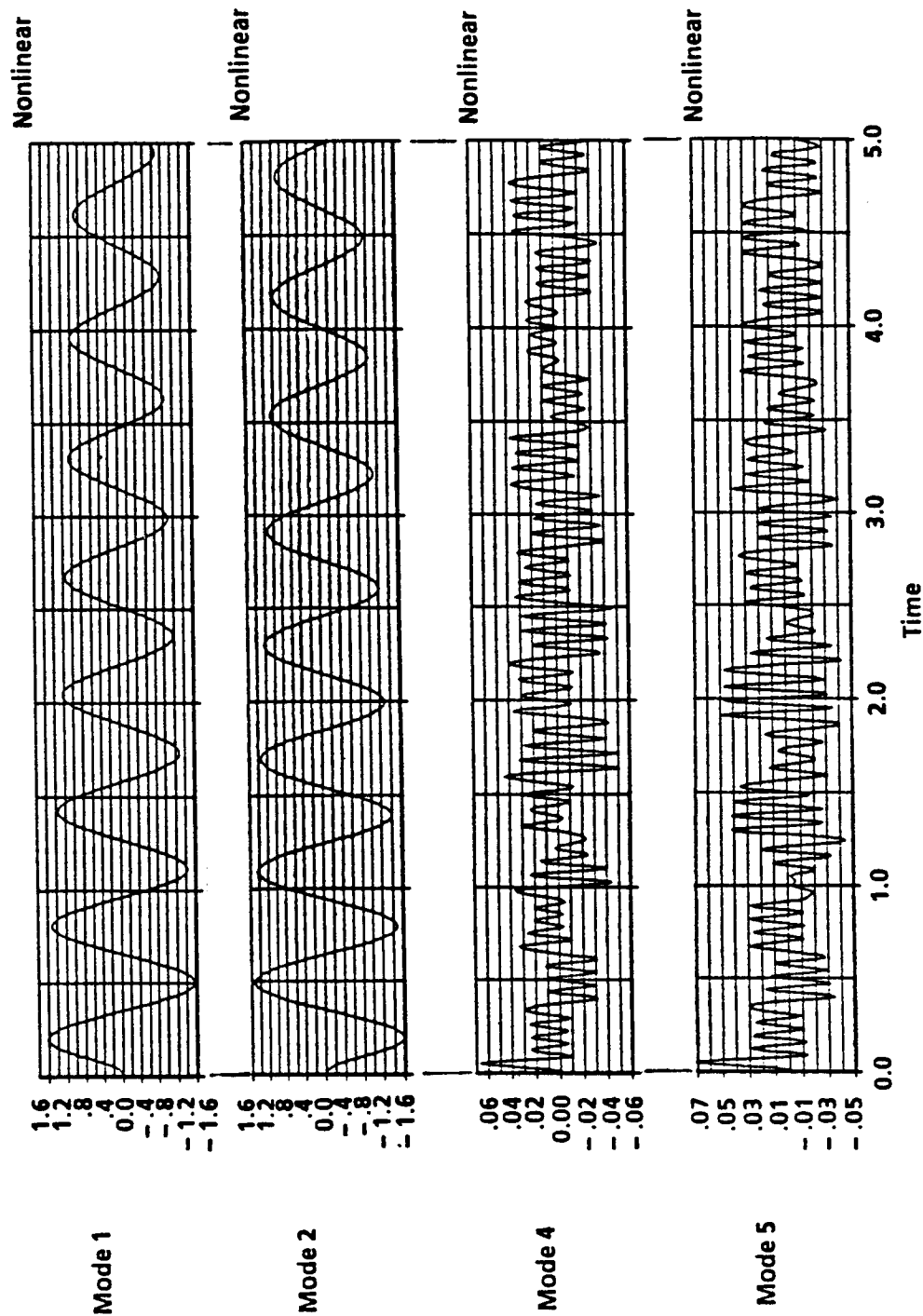


Figure 20

3D Rockwell Truss with 1% Modal Damping

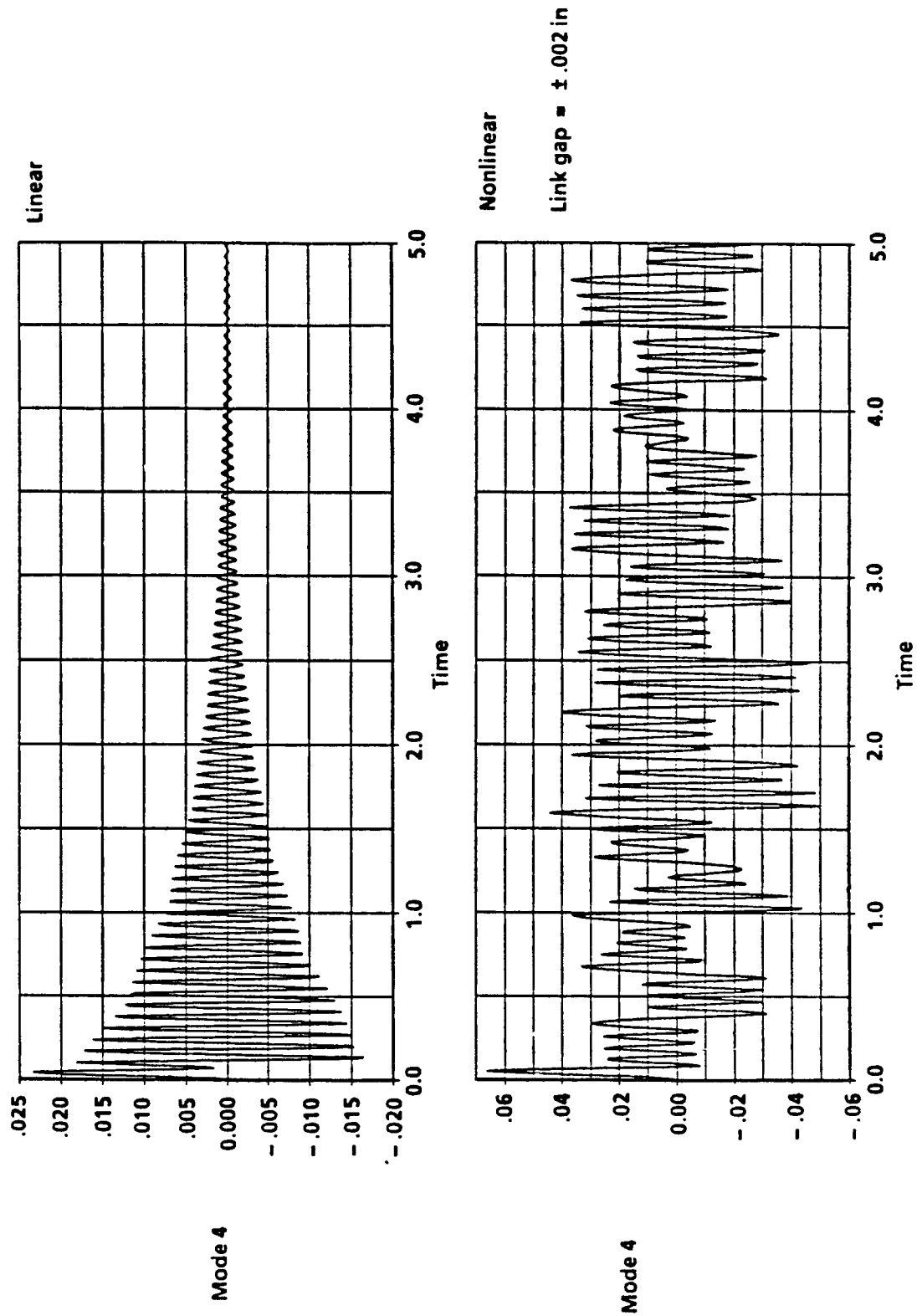


Figure 21

SUMMARY

The transient analysis of trusses having nonlinear joints can be accomplished using the residual force technique. The current technique assumes that the truss links are axial load carrying members only and that the joints have arbitrary force map characterizations. The technique utilizes a "link" concept which has four basic advantages. First, substantial size reduction of the equations of motion is obtained even before modal extraction. Second, numerical difficulties are avoided since the inherently stiff internal degrees of freedom of the links are not monitored. Third, stable integration is achieved by transforming the force maps of the joints to residual force maps of the links. And fourth, direct tests on the links can be performed to validate the analytical assumptions.

The technique was applied a two degree of freedom spring mass system, a four bay planar truss, and an actual ten bay deployable truss at MARSHALL. Joints chosen for analysis were the nonlinear gap joints and the linear Voigt joints. Results from the nonlinear gap analyses generally indicate that coupling between the modes can display some interesting effects during free vibration. One particularly interesting effect was that the damping of the structure appeared to be higher than could be accounted for from modal damping alone. Energy transferral from the lower to the higher modes was found to exist as a result of the modal coupling. The apparently increased damping was due to the fact that the energy transferred to the higher modes is inherently dissipated more quickly. Another interesting phenomenon was that the lower modes could drive the higher modes even during free vibration and that these modes could display a rather large quasi-steady state behavior even when modal damping was present. Gaps were also found to increase the amplitude and period of the free vibration response as expected.

Future work will further examine the effects of modal truncation and residual flexibility that were proposed in the residual force method. Also, other joint nonlinearities will be studied and their effects on the free and forced response of a joint dominated truss determined. Comparison of the analysis predictions with test results also needs to be performed before the residual force technique and truss modeling assumptions can be substantiated.